Examples of Solving $Cm$ Cons*

Solving C-1 from Sample $Cm$ Cryptarithmetic

* "Cm Cons" means “cipher constructions in The Cryptogram” -- the bi-monthly publication for members of the American Cryptogram Association (ACA) -- [www.cryptogram.org](http://www.cryptogram.org)
Examples of Solving

This series shows specific examples of solving ACA ciphers. It tries to give successive hints of what to look at, then follows through by using each hint, building to the solution.

Try to solve the cipher on your own, using as many hints as you need, or just read along.

Please report errors or send suggestions to nudge@cryptogram.org
References

• The ACA and You, Ch. 4, How to Solve a Problem in *The Cryptogram*.

• An Approach to Cryptarithms, by FIDDLE, 1963.
What is a Cryptarithm?

A cryptarithm is an arithmetic operation in which letters have been used to represent the digits. Use the rules of arithmetic to infer what digit each letter represents. Each cryptarithm has a unique solution.

Cryptarithms in $Cm$ are written in a single line to conserve space.
Getting started on a Cryptarithm

Start by rewriting the arithmetic in normal form. Then learn facts about the digits. E.g.,

• What represents zero, one, or nine?
• What cannot represent zero?
• What digit is greater than another?
• What set of digits can each letter represent?
Solving C-1 from Sample $Cm$

C-1. Division. (Two words, 9-0) ZIP
SPOILED ÷ SOLE = DYO; - SPMII = IDNED; - IESPO = MPIL

What does the first line tell us?

Cipher ID: C-1
Type: Division
Key: the letters will spell out two words when listed in order from 9 to 0 (9876543210)
Created by ACA member ZIP
It helps to rewrite a cryptarithm in traditional format, aligning digits correctly. Always a good first step.

A three-digit quotient, but only two subtractions?

\[
\begin{align*}
\text{SPOILED} & \div \text{SOLE} = \text{DYO}; - \text{SPMII} = \text{IDNED}; - \text{IESPO} = \text{MPIL} \\
\end{align*}
\]

Write it in traditional form as far as the first subtraction. What does this reveal?

\[
\begin{array}{c}
\text{DYO} \\
\text{SOLE} \) \text{SPOILED} \\
\text{SPMII} \\
\text{IDNED} \\
\end{array}
\]

9876543210

(2 wds)
Solving C-1 from Sample Cm

It helps to rewrite a cryptarithm in traditional format, aligning digits correctly. Always a good first step.

A three-digit quotient, but only two subtractions?

SPOILED ÷ SOLE = DYO; - SPMII = IDNED; - IESPO = MPIL

An essential clue is given by ED brought down on the right side of the first difference. It tells us that SOLE is larger than IDNE, thus Y=0 and S > I.
Solving C-1 from Sample Cm

It helps to rewrite a cryptarithm in traditional format, aligning digits correctly. Always a good first step.

This is the complete traditional form of our division.

\[
\begin{array}{c}
\text{SPOILED} \\
\text{SOLE) } \\
\hline
\text{SPMII} \\
\text{IDNED} \\
\text{IESPO} \\
\hline
\text{MPIL}
\end{array}
\]

Fact 1: \( S > I \). (Y=0 is recorded at left)

What else can we say about the letters that represent the digits?
Solving C-1 from Sample $Cm$

Fact 2:

$$O > P \text{ (SOLE > SPOI, or else quotient would have 4 digits)}$$

\[
\begin{array}{cccc}
\text{DYO} \\
\hline
\text{SOLE) SPOILED} \\
\text{SPMII} \\
\hline
\text{IDNED} \\
\text{IESPO} \\
\hline
\text{MPIL} \\
\end{array}
\]

$$9876543210$$

$$Y \text{ (2 wds)}$$
Solving C-1 from Sample $C_m$

Fact 3:

\[ D > O \text{ (because } SPMII > IESPO) \]

\[ S > I, \ O > P, \ D > O \]

\[ \text{SOLE) SPOILED} \]
\[ \text{--SPMII--} \]
\[ \text{IDNED} \]
\[ \text{--IESPO--} \]
\[ \text{MPIL} \]

9876543210

Y (2 wds)
Solving C-1 from Sample $C_m$

Fact 4:

$O > M; O > I$ ($O - M = I$ in first subtraction)

$S > I, O > P, D > O, O > M, O > I$

9876543210

Y (2 wds)
Solving C-1 from Sample Cm

Fact 5:
D > E; D > M  (D – E = M in second subtraction)

S > I, O > P, D > O, O > M, O > I, D > E, D > M
Solving C-1 from Sample $C_m$

Fact 6:

\[ D > L \quad (D - O = L \text{ in second subtraction, rightmost digit}) \]

\[
\begin{array}{c}
\text{DYO} \\
\text{SOLE) SPOILED} \\
\text{-SPMII} \\
\text{IDNED} \\
\text{-IESPO} \\
\text{MPIL}
\end{array}
\]

\[ S > I, \quad O > P, \quad D > O, \quad O > M, \quad O > I, \quad D > E, \quad D > M, \quad D > L \]

9876543210

Y (2 wds)
Solving C-1 from Sample $Cm$

Fact 7:

D=9  (I-I=D in first subtraction.  D cannot be zero, so the column to the right must have needed a borrow, forcing this column to borrow from the one on the left, which means D must be 9.)  [also, since D=9, no longer useful to list facts like D > L.]

\[
\begin{array}{c}
\text{DYO} \\
\text{SOLE) SPOILED} \\
\text{-SPMII} \\
\text{IDNED} \\
\text{-IESPO} \\
\text{MPIL}
\end{array}
\]

$S > I, \ O > P, \ O > M, \ O > I$

9876543210
D Y (2 wds)
Solving C-1 from Sample \( C_m \)

Fact 8:

\[ I > L \quad \text{(because L-I in first subtraction caused a borrow)} \]

\[ S > I, \; O > P, \; O > M, \; O > I, \; I > L \]
Solving C-1 from Sample \( C_m \)

Fact 9:

\[ S > M \] (because final remainder must be < SOLE)

\[ S > I, \ O > P, \ O > M, \ O > I, \ I > L, \ S > M \]
Solving C-1 from Sample $C_m$

In second product, O x E ends with O. How might this occur?

$S > I$, $O > P$, $O > M$, $O > I$, $I > L$, $S > M$

DYO
SOLE) SPOILED
-SPMII
-IESPO
MPIL

9876543210
D Y (2 wds)
Solving C-1 from Sample *Cm*

In second product, O x E ends with O. How might this occur? Maybe O=0? Any value of E times O would still end in O.

\[
\begin{array}{c}
\text{S} > \text{I}, \quad \text{O} > \text{P}, \quad \text{O} > \text{M}, \quad \text{O} > \text{I}, \quad \text{I} > \text{L}, \quad \text{S} > \text{M}
\end{array}
\]

\[
\begin{array}{c}
\text{DYO} \\
\text{SOLE)} \quad \text{SPOILED} \\
\text{SPMII} \\
\text{IDNED} \\
\text{IESPO} \\
\text{MPIIL}
\end{array}
\]

9876543210
D Y (2 wds)
In second product, O × E ends with O. How might this occur?
Maybe O=0? Any value of E times O would still end in O.
No, that can’t work.
Y is already 0. O must be some other digit.
Also, in the second subtraction, D – O did not equal D.

S > I, O > P, O > M, O > I, I > L, S > M
Solving C-1 from Sample $Cm$

In second product, $O \times E$ ends with $O$. How might this occur? Maybe $E=1$? Any value of $O$ times $E$ would still end in $O$.

$$S > I, \ O > P, \ O > M, \ O > I, \ I > L, \ S > M$$

9876543210
D        Y (2 wds)
Solving C-1 from Sample Cm

In second product, O x E ends with O. How might this occur?
Maybe E=1? Any value of O times E would still end in O.
No, that can’t work.

In the first product, D x E did not end with D.

\[
\begin{array}{c}
DYO \\
SOLE) SPOILED \\
-SPMII \\
IDNED \\
-IESPO \\
MPIL
\end{array}
\]

\[S > I, \ O > P, \ O > M, \ O > I, \ I > L, \ S > M\]
Solving C-1 from Sample $Cm$

In second product, $O \times E$ ends with $O$. How might this occur? Maybe $O=5$ and $E$ is odd? All products would end with 5.

$S > I, \ O > P, \ O > M, \ O > I, \ I > L, \ S > M$

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9876543210

D Y (2 wds)
Solving C-1 from Sample \( Cm \)

In second product, O \( \times \) E ends with O. How might this occur? Maybe O=5 and E is odd? All products would end with 5. This might be possible. We can further limit the cases.

\( Y=1, \ D=9, \ O=5, \) so E could only be 3 or 7.

\[
\begin{array}{c}
\text{S} > \text{I}, \ \text{O} > \text{P}, \ \text{O} > \text{M}, \ \text{O} > \text{I}, \ \text{I} > \text{L}, \ \text{S} > \text{M}
\end{array}
\]
Solving C-1 from Sample \( Cm \)

In second product, \( O \times E \) ends with \( O \). How might this occur? Maybe \( E=6 \) and \( O \) is even? (e.g., \( 2 \times 6 = 12, \ 4 \times 6 = 24 \))

\[
\begin{array}{c}
\text{1.} \\
\text{2.} \\
\text{3.} \\
\text{4.} \\
\text{5.} \\
\text{6.} \\
\text{7.} \\
\text{8.} \\
\end{array}
\]

\[
\begin{array}{c}
\text{S > I, O > P, O > M, O > I, I > L, S > M}
\end{array}
\]
Solving C-1 from Sample *Cm*

In second product, O x E ends with O. How might this occur? Maybe E=6 and O is even? (e.g., 2 x 6 = 12, 4 x 6 = 24) This might be possible. And we can further limit the cases. E=6, and we ruled out O=0, so O could only be 2, 4, or 8.

S > I, O > P, O > M, O > I, I > L, S > M

9876543210
D Y (2 wds)
Solving C-1 from Sample $Cm$

Summarizing:
Maybe $O=5$ and $E$ is 3 or 7.
Maybe $E=6$ and $O$ is 2, 4, or 8.
List all the combinations. Each column is a possible set of values.

\[
\begin{array}{cccccc}
\text{S} & \text{I} & \text{O} & \text{P} & \text{O} & \text{M} \\
\text{S} & \text{I} & \text{E} & \text{L} & \text{M} & \text{P} \\
\text{O} & 5 & 5 & 2 & 4 & 8 \\
\text{E} & 3 & 7 & 6 & 6 & 6 \\
\end{array}
\]
Solving C-1 from Sample Cm

Add a line for I. D x E ends with I, so fill in values for 9 x E in each column. E.g., in column aa, 9 x 3 = 27, so enter 7 as value for I.

But wait! Columns aa, cc, and dd do not uphold O > I. Throw them out!

\[
\begin{array}{cccccc}
\text{aa} & \text{bb} & \text{cc} & \text{dd} & \text{ee} \\
0 & 5 & 5 & 2 & 4 & 8 \\
E & 3 & 7 & 6 & 6 & 6 \\
I & 7 & 3 & 4 & 4 & 4 \\
\text{x} & \text{x} & \text{x} & \text{x} \\
\end{array}
\]

S > I, O > P, O > M, O > I, I > L, S > M
Solving C-1 from Sample Cm

Enter a line for L. In the second subtraction, \( L = D - O \). Each value for L is calculated as \( 9 - O \). AHA! Values in bb do not uphold \( I > L \). Throw ‘em out!

\[
\begin{array}{cccccc}
S & > & I, & O & > & P, & O & > & M, & O & > & I, & I & > & L, & S & > & M \\
\end{array}
\]

\[
\begin{array}{cccccc}
aa & bb & cc & dd & ee \\
O & 5 & - & - & 8 \\
E & 7 & - & - & 6 \\
I & 3 & - & - & 4 \\
L & 4 & - & - & 1 \\
\end{array}
\]

9876543210
D        Y (2 wds)
Solving C-1 from Sample $Cm$

Only column ee is left. These values must be correct. But first… Let’s create a line for M. In the first subtraction, $M = O – I – 1$ (subtract 1 because the column to the right borrows from this column).

Record all these values on the left.

\[ \begin{array}{cccccc}
\text{aa} & \text{bb} & \text{cc} & \text{dd} & \text{ee} \\
0 & -- & -- & -- & 8 \\
E & -- & -- & -- & 6 \\
I & -- & -- & -- & 4 \\
L & -- & -- & -- & 1 \\
M & -- & -- & -- & 3 \\
\end{array} \]

\[ \begin{array}{cccccc}
\text{DYO} & \text{SOLE} & \text{SPOILED} & \text{SPMII} & \text{IDNED} & \text{IESPO} & \text{MPIL} \\
9876543210 \\
\end{array} \]

DO E IM LY (2 wds)
Solving C-1 from Sample $Cm$

Just a few digits remaining to calculate and record.

$E - P = I$, so $P$ must be 2.
$N - S = P \times 2$, so $N=7$ and $S=5$.

\[\begin{array}{cccccc}
\text{DYO} & \text{SOLE} & \text{SPOILED} \\
\hline
\text{SPMII} & \text{IDNED} \\
\text{IESPO} & \text{MPIL} \\
\end{array}\]

$S > I$, $O > P$, $O > M$, $O > I$, $I > L$, $S > M$

\[
\begin{array}{cccccc}
aa & bb & cc & dd & ee \\
O & - & - & - & - & 8 \\
E & - & - & - & - & 6 \\
I & - & - & - & - & 4 \\
L & - & - & - & - & 1 \\
M & - & - & - & - & 3 \\
\end{array}
\]

DONESIMPLY (2 wds)
Solving C-1 from Sample Cm

All done!

Record the solution so you could later submit it for credit.
C-1 DONESIMPLY

S > I, O > P, O > M, O > I, I > L, S > M

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<td>M</td>
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<td>3</td>
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</tbody>
</table>

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DYO
SOLE) SPOILED
-SPMII
IDNED
-IESPO
MPIL

9876543210
DONESIMPLY (2 wds)
Thank you. Try another. Try the ACA!

The American Cryptogram Association (ACA) is a non-profit organization dedicated to promoting the hobby and art of cryptanalysis – learning to break ciphers. And we write ciphers, too. Our Sample Issue and all its solution tutorials are available on our website:

www.cryptogram.org/resource-area/sample-issue-cryptogram/